2.8 Algebra

DSC-1D **BS:404**

Theory: 4 credits and Practicals: 1 credits
Theory: 4 hours /week and Practicals: 2 hours /week

Objective: The course is aimed at exposing the students to learn some basic algebraic structures like groups, rings etc.

Outcome: On successful completion of the course students will be able to recognize algebraic structures that arise in matrix algebra, linear algebra and will be able to apply the skills learnt in understanding various such subjects.

Unit- I

Groups: Definition and Examples of Groups- Elementary Properties of Groups-Finite Groups; Subgroups -Terminology and Notation -Subgroup Tests - Examples of Subgroups Cyclic Groups: Properties of Cyclic Groups - Classification of Subgroups Cyclic Groups-Permutation Groups: Definition and Notation -Cycle Notation-Properties of Permutations -A Check Digit Scheme Based on D_5.

Unit- II

Isomorphisms ; Motivation- Definition and Examples -Cayley's Theorem Properties of Isomorphisms -Automorphisms-Cosets and Lagrange's Theorem Properties of Cosets 138 - Lagrange's Theorem and Consequences-An Application of Cosets to Permutation Groups -The Rotation Group of a Cube and a Soccer Ball -Normal Subgroups and Factor Groups ; Normal Subgroups-Factor Groups -Applications of Factor Groups -Group Homomorphisms - Definition and Examples -Properties of Homomorphisms -The First Isomorphism Theorem.

Unit- III

Introduction to Rings: Motivation and Definition -Examples of Rings -Properties of Rings -Subrings -Integral Domains : Definition and Examples –Characteristics of a Ring -Ideals and Factor Rings; Ideals -Factor Rings -Prime Ideals and Maximal Ideals.

Unit- IV

Ring Homomorphisms: Definition and Examples-Properties of Ring- Homomorphisms -The Field of Quotients Polynomial Rings: Notation and Terminology.
Text:

- Joseph A Gallian, *Contemporary Abstract algebra (9th edition)*

References:

- Bhattacharya, P.B Jain, S.K.; and Nagpaul, S.R,*Basic Abstract Algebra*
- Fraleigh, J.B, *A First Course in Abstract Algebra.*
- Herstein, I.N, *Topics in Algebra*
- Robert B. Ash, *Basic Abstract Algebra*
- I Martin Isaacs, *Finite Group Theory*
- Joseph J Rotman, *Advanced Modern Algebra*

2.8.1 Practicals Question Bank

Algebra

Unit-I

1. Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group but that $\{1, 2, 3, 4\}$ under multiplication modulo 5 is a group.

2. Let G be a group with the property that for any x, y, z in the group, $xy = zx$ implies $y = z$. Prove that G is Abelian.

3. Prove that the set of all 3×3 matrices with real entries of the form

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

   is a group under multiplication.

4. Let G be the group of polynomials under addition with coefficients from Z_{10}. Find the orders of $f(x) = 7x^2 + 5x + 4, g(x) = 4x^2 + 8x + 6$, and $f(x) + g(x)$

5. If a is an element of a group G and $|a| = 7$, show that a is the cube of some element of G.

6. Suppose that $\langle a \rangle, \langle b \rangle$ and $\langle c \rangle$ are cyclic groups of orders 6, 8, and 20, respectively. Find all generators of $\langle a \rangle, \langle b \rangle$, and $\langle c \rangle$.

7. How many subgroups does Z_{20} have? List a generator for each of these subgroups.

8. Consider the set $\{4, 8, 12, 16\}$. Show that this set is a group under multiplication modulo 20 by constructing its Cayley table. What is the identity element? Is the group cyclic? If so, find all of its generators.

9. Prove that a group of order 4 cannot have a subgroup of order 3.

10. Determine whether the following permutations are even or odd.
    a. (135)
    b. (1356)
    c. (13567)
    d. (12)(134)(152)
    c. (1243)(3521).

Unit-II

11. Show that the mapping $a \longrightarrow \log_{10} a$ is an isomorphism from R^+ under multiplication to R under addition.

12. Show that the mapping $f(a + bi) = a - bi$ is an automorphism of the group of complex numbers under addition.

13. Find all of the left cosets of $\{1, 11\}$ in $U(30)$.

14. Let C^* be the group of nonzero complex numbers under multiplication and let $H = \{a+bi \in C^*/a^2 + b^2 = 1\}$. Give a geometric description of the coset $(3 + 4i)H$. Give a geometric description of the coset $(c + di)H$.

15. Let $H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} /a, b, d \in R, ad \neq 0 \right\}$. Is H a normal subgroup of $GL(2, R)$?

16. What is the order of the factor group $\frac{Z_{60}}{\langle 5 \rangle}$?

17. Let $G = U(16), H = \{1, 15\}$, and $K = \{1, 9\}$. Are H and K isomorphic? Are G/H and G/K isomorphic?

18. Prove that the mapping from R under addition to $GL(2, R)$ that takes x to

$$\begin{bmatrix} cosx & sinx \\ -sinx & cosx \end{bmatrix}$$

is a group homomorphism. What is the kernel of the homomorphism?

19. Suppose that f is a homomorphism from $Z30$ to $Z30$ and $Kerf = \{0, 10, 20\}$. If $f(23) = 9$, determine all elements that map to 9.

20. How many Abelian groups (up to isomorphism) are there
a. of order 6?
b. of order 15?
c. of order 42?
d. of order pq, where p and q are distinct primes?
e. of order pqr, where p, q, and r are distinct primes?

Unit-III

21. Let $M_2(Z)$ be the ring of all 2×2 matrices over the integers and let $R = \left\{ \begin{pmatrix} a & a \\ b & b \end{pmatrix} /a, b \in Z \right\}$
Prove or disprove that R is a subring of $M_2(Z)$.

22. Suppose that a and b belong to a commutative ring R with unity. If a is a unit of R and $b^2 = 0$, show that $a + b$ is a unit of R.

23. Let n be an integer greater than 1. In a ring in which $x^n = x$ for all x, show that $ab = 0$ implies $ba = 0$.

24. List all zero-divisors in Z_{20}. Can you see a relationship between the zero-divisors of Z_{20} and the units of Z_{20}?

25. Let a belong to a ring R with unity and suppose that $a^n = 0$ for some positive integer n. (Such an element is called nilpotent.) Prove that $1 - a$ has a multiplicative inverse in R.

26. Let d be an integer. Prove that $Z[\sqrt{d}] = \{a + b\sqrt{d}/a, b \in Z\}$ is an integral domain.

27. Show that Z_n has a nonzero nilpotent element if and only if n is divisible by the square of some prime.

28. Find all units, zero-divisors, idempotents, and nilpotent elements in $Z_3 \bigoplus Z_6$.

29. Find all maximal ideals in
 a. Z_8.
 b. Z_{10}.
 c. Z_{12}.
 d. Z_n.

30. Show that $R[x]/\left\langle x^2+1\right\rangle$ is a field.

Unit-IV

31. Prove that every ring homomorphism f from Z_n to itself has the form $f(x) = ax$, where $a^2 = a$.

32. Prove that a ring homomorphism carries an idempotent to an idempotent.

33. In Z, let $A = \langle 2\rangle$ and $B = \langle 8\rangle$. Show that the group A/B is isomorphic to the group Z_4 but that the ring A/B is not ring-isomorphic to the ring Z_4.

34. Show that the number $9,897,654,527,609,805$ is divisible by 99.

35. Show that no integer of the form $111,111,111,...,111$ is prime.

36. Let $f(x) = 4x^3 + 2x^2 + x + 3$ and $g(x) = 3x^4 + 3x^3 + 3x^2 + x + 4$, where $f(x), g(x) \in Z_5[x]$. Compute $f(x) + g(x)$ and $f(x).g(x)$.

37. Let $f(x) = 5x^4 + 3x^3 + 1$ and $g(x) = 3x^2 + 2x + 1$ in $Z_7[x]$.Determine the quotient and remainder upon dividing $f(x)$ by $g(x)$.

38. Let $f(x)$ belong to $Z_p[x]$. Prove that if $f(b) = 0$, then $f(b^p) = 0$.

39. Is the mapping from Z_{10} to Z_{10} given by $x \to 2x$ a ring homomorphism?

40. Determine all ring homomorphisms from Z to Z.

Skill Enhancement Course - II- B.Sc., II YEAR, IV Semester
FOR ALL SCIENCE FACULTY DEPARTMENTS
MULTIMEDIA AND APPLICATIONS

Credits: 2 Theory: 2 hours/week
Marks - 50

Unit - I FONTS AND IMAGES

1.1.Multimedia: Introduction to multimedia, components, uses of multimedia, Multimedia applications, virtual reality.

1.2.Text: Fonts and Faces, Using Text in Multimedia, Font Editing and Design Tools, Hypermedia & Hypertext.

1.3.Images: Still Images – bitmaps, vector drawing, 3D drawing and rendering, natural, light and colors, computerized colors, color palettes, image file formats.

Unit – II AUDIO AND VIDEO

2.1.Sound: Digital Audio, MIDI Audio, MIDI vs Digital Audio, Audio File Formats.

2.2Video: How video works, analog video, digital video, video file formats, video shooting and editing.

2.3Animation: Principle of animations, animation techniques, animation file formats.

References:

1. Tay Vaughan, ―Multimedia: Making it work‖, TMH, Eighth edition.2011

2. Ralf Steinmetz and KlaraNaharstedt, ―Multimedia: Computing, Communications Applications‖, Pearson.2012

3. Keyes, ―Multimedia Handbook‖, TMH,2000.
4. K. Andleigh and K. Thakkar, ―Multimedia System Design‖, PHI.2013